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On the Polynomial Inequalities in the Weighted Lebesgue Spaces

Let \mathbb{C} be a complex plane, and let $G \subset \mathbb{C}$ be a bounded Jordan region, $L := \partial G$, $\Omega := \overline{\mathbb{C}} \setminus \overline{G}$. Denote by \wp_n a class of arbitrary algebraic polynomials $P_n(z)$ of degree at most $n \in \mathbb{N} := \{1, 2, \dots\}$.

For any $0 < p \leq \infty$ and for a rectifiable Jordan curve L , set \mathcal{L}

$$\|P_n\|_p : = \|P_n\|_{\mathcal{L}_p(h,L)} := \left(\int_L h(z) |P_n(z)|^p |dz| \right)^{1/p}, \quad 0 < p < \infty,$$
$$\|P_n\|_\infty : = \|P_n\|_{\mathcal{L}_\infty(1,L)} := \max_{z \in L} |P_n(z)|, \quad p = \infty.$$

Let $\{z_j\}_{j=1}^m$ be a fixed system of distinct points on curve L . Consider a Jacobi weight function $h(z)$ defined on a bounded region $\dot{G}_1 \supset G$ as follows:

$$h(z) := h_0(z) \prod_{j=1}^m |z - z_j|^{\gamma_j}, \quad (1)$$

where $\gamma_j > -1$, for all $j = 1, 2, \dots, m$, and h_0 is uniformly separated from zero.

Many problems of the approximation theory, theory of polynomials and others, stimulate the study of the following inequality:

$$\|P_n\|_{\mathcal{L}_\infty} \leq c \mu_n(L, h, p) \|P_n\|_{\mathcal{L}_p(h,L)}, \quad (2)$$

where the constant $c = c(L, p) > 0$ does not depend of n and P_n , and $\mu_n(L, h, p) \rightarrow \infty$, as $n \rightarrow \infty$, depending on geometrical properties of the curve L and the weight function h in neighborhoods of $\{z_j\}_{j=1}^m$.

The classical results of (2)-type can be found in [5], [8]. The estimation of (2)-type was investigated by [6], [7, pp.122-133], [4], [1]-[3] and others.

In this work, we investigate the condition “pay off” singularity of the curve and the weight function, whose estimation of (2) coincides with an estimation when the boundary curve and weight functions have no singularities. We also consider the cases when the conditions “pay off” singularity of the curve and the weight function do not hold.

REFERENCES

- [1] Abdullayev, F. G., Özkartepe N.P., *On the Behavior of the Algebraic Polynomial in Unbounded Regions with Piecewise Dini-Smooth Boundary*, Ukrainian Mathematical Journal, 66 (5), (2014), pp.579-597.
- [2] Abdullayev F.G., Özkartepe N.P., *On the growth of algebraic polynomials in the whole complex plane*, Journal of Korean Math. Soc., 52, 4 (2015), 699-725.
- [3] Abdullayev F.G., Özkartepe N.P., *Uniform and pointwise polynomial inequalities in regions with cusps in the weighted Lebesgue space*, Jaen Journal on Approximation, Vol.7, No:2, (2015), pp.231-261.
- [4] Andrievskii V.V., *Weighted Polynomial Inequalities in the Complex Plane*, Journal of Approximation Theory, Vol. 164 Issue 9, (2012), pp. 1165-1183.
- [5] Jackson D., *Certain problems on closest approximations*, Bull. Amer. Math. Soc., 39, (1933), pp.889-906.
- [6] Mamedhanov D.I., *Inequalities of S.M.Nikol'skii type for polynomials in the complex variable on curves*, Soviet Math.Dokl., 15, (1974), pp.34-37.

- [7] Nikol'skii S.M., *Approximation of Function of Several Variable and Imbedding Theorems*, Springer-Verlag, New-York, 1975.
- [8] Szegő G. and Zigmund A., *On certain mean values of polynomials*, J.Anal. Math., 3, (1954), pp.225-244.

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Representation formulas for Hardy space functions through the Cuntz relations and new interpolation problems

We introduce connections between the Cuntz relations and the Hardy space H^2 of the open unit disk D . We then use them to solve a new kind of multipoint interpolation problem in H^2 , where for instance, only a linear combination of the values of a function at given points is preassigned, rather than the values at the points themselves. We also consider the case of rational functions when no metric constraints are given.

The talk is based on the papers

REFERENCES

- [1] D. Alpay, P. Jorgensen, I. Lewkowicz, and D. Volok. A new realization of rational functions, with applications to linear combination interpolation. *Complex Variables and Elliptic Equations*, vol. 61 (2016), no. 1, 42-54.
- [2] D. Alpay, P. Jorgensen, I. Lewkowicz and I. Marziano. Representation formulas for Hardy space functions through the Cuntz relations and new interpolation problems. In: *Multiscale signal analysis and modeling* (editors Xiaoping Shen and Ahmed Zayed), Lecture Notes in Electrical Engineering (Springer), 2013, pp. 161-182.

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Schottky Implies Poincare in all genus bigger than 5

We shall show how Schottky type relations imply the relations of Poincare thus allowing us to write a set of $(g - 3)(g - 2)/2$ functionally independent Schottky type relations in a neighborhood of the diagonal matrices which then contains a component of the Jacobians.

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Translation invariant probability measures on the space of entire functions

20 years ago Benjy Weiss constructed a collection of non-trivial translation invariant probability measures on the space of entire functions. In this talk we will present a construction of such a measure, and give upper and lower bounds for the possible growth of entire functions in the support of such a measure. We will also discuss "uniformly recurrent" entire functions, their connection to such constructions, and their possible growth.

The talk is based on a joint work with Mikhail Sodin.

Vladimir Gol'dshtein

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Spectral properties of Laplace operator and hyperbolic geometry

The talk is devoted to connections between hyperbolic geometry of bounded simply connected planar domains and spectral properties of p -Laplace operator ($1 < p \leq 2$) in the cases of Dirichlet and Neumann boundary problems. The classical results by L. E. Payne and H. F. Weinberger give the lower estimates of the first non-trivial eigenvalue of the Neumann Laplacian in convex domains in terms of its domains. The Nikodim's type examples show that in the general case of simply connected planar domains the first non-trivial eigenvalue can not be estimated in terms of its diameters. We suggest the estimates in terms of hyperbolic radius for a large class of bounded non convex domains with some additional restrictions on the hyperbolic geometry that we call a conformal regularity. This new class of domains includes quasicircles (images of the unit disc under quasiconformal homeomorphism of the plane). For example, it means that boundaries of domains can have any Hausdorff dimension between 1 and 2. In our opinion the hyperbolic geometry represents more appropriate language for description of spectral behavior of the Laplace operator than the Euclidian one.

Joint work with Alexander Ukhlov.

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Uniform Bounds on Zero Sets of Smooth Functions

The main theorem of this talk is the following: any smooth, real-valued function of finite vanishing order has a zero set of co-dimension at least 1 (assuming the function is not identically 0). This result is classical if the function in question is analytic or, by work of Hardt-Simon, if it smooth and satisfies a second order elliptic equation. On the other hand, the result is false without the finite vanishing order assumption due to the existence of bump functions (and more generally that any closed subset of \mathbb{R}^n is the zero set of some smooth function).

A consequence of our main result is the first proof that on any smooth compact Riemannian manifold (M, g) , finite linear combinations of Laplace eigenfunctions must have co-dimension 1 zero sets. We actually prove more generally upper bounds on the Hausdorff measures of zero sets of solutions to the heat equation on (M, g) with L^2 initial data. I will explain how to combine these results with work of Gromov, Guth, and Marques-Neves to get new lower bounds on the Hausdorff measure of zeros sets for finite linear combinations of Laplace eigenfunctions.

Joint work with Tom Beck and Spencer Hughes.

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Regularity and modulus of continuity for space-filling curves

The classical example of Peano gives a continuous map from the unit interval so that the image covers a square. Similarly, there exist continuous maps from the unit interval into \mathbb{R}^n for $n \geq 3$ so that the image of the interval covers a cube. It is then natural to ask how regular such maps can be. Towards this, we define a family of energies and a family of moduli of continuity. Based on these we state rather sharp conditions that rule out this type of pathological behavior. The techniques trace back to studies on univalent functions.

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Real analytic Frechét algebras

We show that there are function algebras on \mathbb{C} , closed under the topology of uniform convergence on compacta, containing the entire holomorphic functions as a proper subalgebra, such that every function in the algebra is real analytic, with infinite radius of convergence. This construction lifts to \mathbb{C}^n , to some Stein spaces, and to some CR manifolds, the last if interpreted properly. Generators and ideal structure are examined. We show that functions in these algebras of low order of growth must be holomorphic. Several open questions will be discussed at the end.

Nir Lev

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Tiling by translates of a function

A function f on the real line is said to tile by translates along a discrete set Λ if we have

$$\sum_{\lambda \in \Lambda} f(x - \lambda) = 1$$

almost everywhere. Which functions can tile by translates, and what can be said about the translation set Λ ? I will survey the subject and discuss some recent results joint with Mihail Kolountzakis.

Alexander Logunov

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Estimates for nodal sets of Laplace eigenfunctions

We will discuss the recent progress in understanding zero sets of Laplace eigenfunctions on smooth Riemannian manifolds and a related question by Nadirashvili.

Ron Peled

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Separating signal from noise

Suppose that a sequence of numbers (x_n) (a signal) is transmitted through a noisy channel. The receiver observes a noisy version of the signal, $(x_n + xi_n)$, where the (xi_n) are independent standard Gaussian random variables. Suppose further that the signal is known to come from some fixed space X of possible signals. Is it possible to fully recover the transmitted signal from its noisy version? Is it possible to at least detect that a non-zero signal was transmitted?

We study the case in which signals are infinite sequences and the recovery or detection are required to hold with probability one. We provide conditions on the space X for checking whether detection or recovery are possible and illustrate their applicability on examples related to statistics, harmonic analysis, ergodic theory and probability. Many of our examples exhibit critical phenomena, in which a sharp transition is made from a regime in which recovery is possible to a regime in which even detection is impossible.

Joint work with Nir Lev and Yuval Peres.

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On the Riemann-Hilbert Problem

It is proved the existence of solutions for the Riemann-Hilbert problem in the fairly general settings of arbitrary Jordan domains, measurable coefficients and measurable boundary data. The theorem is formulated in terms of harmonic measure and principal asymptotic values. It is also given the corresponding reinforced criterion for domains with arbitrary rectifiable boundaries stated in terms of the natural parameter and nontangential limits. Furthermore, it is shown that the dimension of the spaces of solutions is infinite.

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Multiplicative chaos measures for a random model of the Riemann zeta function

We study the existence of non-Gaussian multiplicative chaos measures that are constructed over a random field that arises as a model for the statistical behaviour of the Riemann zeta-function over the critical line.

The talk is based on joint work with Christian Webb (Aalto University).

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On a class of starlike functions

Let \mathcal{A} be the class of functions f that are analytic in the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ and are normalized such that $f(0) = f'(0) - 1 = 0$. Also, let \mathcal{S}^* be the class of normalized starlike univalent functions

$$\mathcal{S}^* = \left\{ f \in \mathcal{A} : \operatorname{Re} \left[\frac{zf'(z)}{f(z)} \right] > 0, z \in \Delta \right\}.$$

In the past, the expression

$$\frac{f(z)f''(z)}{[f'(z)]^2} = 1 - \left[\frac{f(z)}{f'(z)} \right]'$$

has attracted considerable attention as a criteria for starlikeness. We give a review of the most characteristic results.

Using this expression and the analytical representation of starlikeness, we define the following operator acting on the functions in \mathcal{A} :

$$I[f](t, z) = t|z|^2 \frac{f(z)}{zf'(z)} + (1-t) \left[1 - \frac{f(z)f''(z)}{[f'(z)]^2} \right] (1 - |z|^2)$$

(t is real and z is in Δ). This operator defines a class of functions depending on a real parameter $t \in [0, 1]$:

$$\mathcal{S}_t = \{f \in \mathcal{A} : \operatorname{Re} I[f](t, z) \geq 0, z \in \Delta\}.$$

For the classes \mathcal{S}_t , we prove the following inclusion property

Theorem 1. $\mathcal{S}_s \subset \mathcal{S}_t \subseteq \mathcal{S}_r = \mathcal{S}^*$ when $0 \leq s < t \leq \frac{2}{3} \leq r \leq 1$.

Theorem 1 indicates that

$$t \in [2/3, 1] \quad \Rightarrow \quad b(t) := \inf \left\{ \operatorname{Re} \frac{f(z)}{zf'(z)} : f \in \mathcal{S}_t, z \in \Delta \right\} = 0.$$

As it turns out, the value of $b(t)$ when $t \in [0, 2/3)$ is rather unexpected:

Theorem 2. $b(0) = -1 + 2 \ln 2 \approx 0.38629 \dots$ and $b(t) = 0$ for $t \in (0, 2/3)$.

At the end, we study the growth estimates, i.e., the lower and upper bound of $|f(z)|$, and conjecture the values for the functions f in \mathcal{S}_t .

Join work with D. Shoikhet.